K26U 0195

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/ Supplementary/Improvement) Examination, April 2026
(2020 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Define uniform continuity.
2. State product theorem.
3. Is every bounded function on [a, b] integrable ?
4. Compute $\Gamma(-1/2)$.
5. Give an example of a metric space which is not complete. **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. If $f : A \to \mathbb{R}$ is a Lipschitz function, prove that f is uniformly continuous on A.
7. Show that if f and g are uniformly continuous on a subset A of $\mathbb{R}$, then f + g is uniformly continuous on A.
8. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be increasing on I. If $c \in I$, prove that f is continuous at c if and only if $j_f(c) = 0$.
9. Prove that every constant function on [a, b] is in $\mathcal{R}[a, b]$.

P.T.O.

10. Suppose that f and g are in $\mathcal{R}[a, b]$. If $f(x) \le g(x)$ for all $x \in [a, b]$, prove that $\int_a^b f \le \int_a^b g$.

11. State first form of fundamental theorem of calculus.

12. Prove that $\int_0^\infty \left[\log\left(\frac{1}{t}\right)\right]^{n-1} dt = \Gamma n$.

13. Prove that $\Gamma(1/2) = \sqrt{\pi}$.

14. Evaluate $\int_0^\infty \frac{dx}{1+x^4}$.

15. Evaluate $\lim\left(\frac{x^n}{1+x^n}\right)$ for $x \in \mathbb{R}$, $x \ge 0$.

16. Determine the radius of convergence of the series $\sum \frac{1}{n^n}$. **(8×2=16)**

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Let I be a closed and bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. If $\in > 0$, prove that there exists a step function $S_\in : I \to \mathbb{R}$ such that $|f(x) - S_\in(x)| < \in$ for all $x \in I$.

18. If $f \in \mathcal{R}[a, b]$, prove that the value of the integral is uniquely determined.

19. If $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], prove that $f \in \mathcal{R}[a, b]$.

20. Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

21. Evaluate $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}$.

22. State and prove Cauchy Hadamard theorem.

23. For any two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ on $\mathbb{R}^2$, define $d_\infty (P_1, P_2) = \sup\{|x_1 - x_2|, |y_1 - y_2|\}$. Prove that d_∞ is a metric on $\mathbb{R}^2$. **(4×4=16)**

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. State and prove continuous extension theorem.

25. State and prove squeeze theorem.

26. Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \cdot \int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\pi\sqrt{2}}{4}$.

27. a) State and prove Weierstrass M test.

b) Discuss the convergence and uniform convergence of the series $\sum \frac{1}{x^2 + n^2}$. **(2×6=12)**