K24U 0061

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Define subspace of a vector space.
2. What is the dimension of the vector space of all 2×3 matrices over R ?
3. State Dimension Theorem.
4. The characteristic roots of a matrix A are 2, 3 and 4. Then find the characteristic roots of the matrix 3A.
5. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$ and $c\,(a_1, a_2) = (ca_1, 0)$. Is V a vector space over R with these operations ? Justify your answer.
7. Prove that the set of all symmetric matrices of order n is a subspace of the vector space of all square matrices of order n.

P.T.O.

8. Check whether the set {(1, –1, 2), (2, 0, 1), (–1, 2, –1)} is linearly independent or not.

9. Give an example of three linearly dependent vectors in R^3 such that none of the three is a multiple of another.

10. Find the rank of matrix A, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

11. Show that rank of a matrix, every element of which is unity, is 1.

12. Show that $T : R^2 \to R^2$ defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ is a linear transformation.

13. Explain the condition for consistency and nature of solution of a non homogeneous linear system of equations AX = B.

14. Let $T : V \to V$ be a linear transformation. Find the range and null space of zero transformation and identity transformation.

15. Prove that the Eigen values of an idempotent matrix are either zero or unity.

16. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

PART – C

Answer **any** 4 questions. **Each** question carries **four** marks.

17. Prove that any intersection of subspaces of a vector space V is a subspace of V.

18. Suppose that $T : R^2 \to R^2$ is linear, T(1,0) = (1,4) and T(1,1) = (2,5). What is T(2,3) ? Is T one-to-one ?

19. Let $T : R^2 \to R^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for R^2 and γ = { (1, 1, 0), (0, 1, 1), (2, 2, 3)}. Compute $[T]_\beta^\gamma$.

20. Under what condition the rank of the following matrix A is 3 ? Is it possible for the rank to be 1 ? Why ? $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$.

21. Solve the system of equations.

$x - 2y + 3z = 0$

$2x + y + 3z = 0$

$3x + 2y + z = 0$

22. Find the eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

23. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ find A^2 using Cayely Hamilton theorem and then find A^3.

PART – D

Answer **any 2** questions. **Each** question carries **six** marks.

24. Prove that the set of all $m \times n$ matrices with entries from a field F is a vector space over F with the operations of matrix addition and scalar multiplication.

25. Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row operations.

26. Find the values of a and b for which the system of equations

$x + y + z = 3$

$x + 2y + 2z = 6$

$x + 9y + az = b$ have

1) no solution;

2) unique solution and;

3) an infinite number of solutions.

27. Using Cayley Hamilton theorem find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.