K26FY1381 Pages: 2 Reg No:.....
Name:.....

Second Semester FYUGP Degree (Reg/Sup/Imp) Examination
April 2026

KU2DSCMAT113 - SET THEORY, NUMBER THEORY, INTEGRAL CALCULUS AND FOURIER SERIES

2024 Admission onwards

Time : 2 hours Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Evaluate $\int \sin^3 x\,dx$, using reduction formula..
2. Evaluate $\int \sin^4 x\,dx$, using reduction formula.
3. Write 1125 as a product of powers of primes.
4. Calculate $LCM(72, 108)$.
5. Write the base 7 expansion of 215.
6. What is the general form of a Fourier series for a function $f(x)$ with period 2π?
7. What is the fundamental period of the function $f(x) = \sin 3x$?
8. Write is the formula for the Fourier coefficient b_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$ of a $2L$ periodic function $f(x)$.

Section B

Answer any 4 questions. Each carry 6 marks.

9. (a) If $a \mid b$ or $a \mid c$ then show that $a \mid bc$, where a, b and c are integers.
   (b) If $a \mid b$ and $b \mid c$ then show that $a \mid c$, where a, b and c are integers.
10. (a) Compute $GCD(108, 60)$ using Euclidean Algorithm.
    (b) Show that if $GCD(a, c) = 1$ and $c \mid ab$, then $c \mid b$.
11. If f is the $mod - 7$ function, compute
    (a) $f(752) + f(793)$
    (b) $f(752 + 793)$
    (c) $f(3 \times 1759)$.

12. Determine the Fourier series coefficient a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ of the function $f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.

13. Obtain the Fourier coefficients a_0 and a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x\right)$ of the 2-periodic function defined by:

$$f(x) = \begin{cases} 0, & \text{when } -1 < x < 0 \\ 1, & \text{when } 0 < x < 1. \end{cases}$$

14. Obtain the Fourier coefficient a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x\right)$ of the function defined by:

$$f(x) = \begin{cases} 0, & \text{when } -2 < x < -1 \\ k, & \text{when } -1 < x < 1 \\ 0, & \text{when } 1 < x < 2 \end{cases}$$

with period $p = 2L = 4$.

Section C

Answer any 2 questions. Each carry 14 marks.

15. Given the sets $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6, 7\}$, and the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
    (a) Find $X \oplus Y$.
    (b) Verify that $|X \cup Y| = |X| + |Y| - |X \cap Y|$.
    (c) Verify De Morgan's Law: $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$ and $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.

16. A survey of 500 television watchers produced the following information. 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games, and 50 do not watch any of the three kinds of games.
    (a) How many people in the survey watch all three kinds of games?
    (b) How many people watch exactly one of the sports?

17. (a) Connect the integrals $\int \sin^m x \cos^n x dx$ and $\int \sin^{m-2} x \cos^n x dx$
    (b) Evaluate $\int \cos^5 x dx$.