



K20U 1537

Reg. No. : .....

Name : .....



V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.)  
Examination, November 2020  
(2017 Admn. Onwards)  
CORE COURSE IN MATHEMATICS  
5B09 MAT : Graph Theory

Time : 3 Hours

Total Marks : 48

PART – A

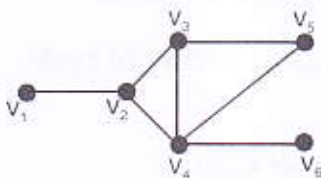
Answer **all 4** questions : (4×1=4)

1. Draw a graph on 4 vertices having a cut vertex. Mark the cut vertices.
2. Sketch 2 isomorphic trees on 4 vertices.
3. Plot a strict digraph on 4 vertices.
4. Sketch a symmetric digraph on 4 vertices.

PART – B

Answer **any 8** questions : (8×2=16)

5. Define a complete graph. Draw the graph  $K_5$ .
6. Picturise all non isomorphic graphs on 3 vertices.
7. If  $e = xy$  is a cut edge of a connected graph  $G$ , prove that there exist vertices  $u$  and  $v$  such that  $e$  belongs to every  $u$ - $v$  path in  $G$ .
8. Find the cut edges and the cut vertices of the graph given below.



P.T.O.



9. Draw a 2 regular graph on 4 vertices and draw one spanning graph of the same.
10. For a connected graph  $G$ , define the terms diameter and eccentricity.
11. Find a covering and a minimal covering for the wheel graph  $W_5$ .
12. Give an example of an Eulerian graph. Explain why it is Eulerian.
13. Explain the terms Directed Walk and Directed Cycle.
14. Define the term tournament. Sketch a tournament on 3 vertices.

## PART – C

Answer **any 4** questions :

(4×4=16)

15. Plot all non isomorphic graphs on 4 vertices.
16. If a simple graph  $G$  is not connected, prove that  $G^c$  is connected.
17. Prove that a graph  $G$  with at least 3 vertices is 2-connected if and only if any two vertices of  $G$  lie on a common cycle.
18. Prove that a graph is a tree if and only if any two distinct vertices are connected by a unique path.
19. For a graph  $G$  on  $n$  vertices, define the terms independence number  $\alpha$  and the covering number  $\beta$  of  $G$ . Further show that  $\alpha + \beta = n$ .
20. Describe Königsberg bridge problem. Represent the problem graphically. Does the problem has a solution ? Explain.

## PART – D

Answer **any 2** questions :

(2×6=12)

21. Show that a graph  $G$  is bipartite if and only if it contains no odd cycle.
  22. Prove that a graph  $G$  with at least three vertices is 2-connected if and only if any two vertices of  $G$  are connected by at least 2 internally disjoint paths.
  23. Establish the claim : A graph is Eulerian if and only if it has odd number of cycle decompositions.
  24. Prove that every tournament contains a directed Hamiltonian path.
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