



K26U 0872

Reg. No. :

Name :

Second Semester B.Sc. AI and ML Degree (C.B.C.S.S. – O.B.E. –
Supplementary) Examination, April 2026
(2023 Admission)
Complementary Elective Course
2C02MAT – AIML : INTEGRATION AND LINEAR ALGEBRA

Time : 3 Hours

Max. Marks : 40

PART – A
(Short Answer)

Answer **all** questions from this Part. Each question carries 1 mark. (6×1=6)

1. State Euler's theorem on homogeneous functions.
2. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \cos(xy)$.
3. Evaluate $\int_0^{\pi/2} \cos^6 x \, dx$.
4. Write the reduction formula for $\int \sin^n x \, dx$.
5. Give an example of a two-dimensional vector space.
6. Define the characteristic equation of a matrix A.

PART – B
(Short Essay)

Answer **any six** questions from this Part. Each question carries 2 marks. (6×2=12)

7. If $u = x^y$, find $\frac{\partial u}{\partial y}$.
8. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} \, dx$.

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9. Evaluate $\int_0^{\pi/6} \cos^4(3\theta) \sin^3(6\theta) d\theta$.
10. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
11. Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$.
12. If $u = \log\left(\frac{x^2}{y}\right)$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.
13. Give an example of a linear transformation from \mathbb{R}^2 to \mathbb{R}^3 .
14. Define the dimension of a vector space. Give an example of a two-dimensional vector space.

PART - C

(Essay)

Answer **any four** questions from this Part. **Each** question carries 3 marks. **(4×3=12)**

15. If $u = F(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
16. If $z = f(x + ct) + \phi(x - ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$.
17. Evaluate $\int_0^{\pi} \frac{\sqrt{1 - \cos x}}{1 + \cos x} \sin^2 x dx$.
18. Show that the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ satisfies the Cayley-Hamilton theorem.
19. Determine the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
20. Determine the nature of the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.



PART – D
(Long Essay)

Answer any two questions from this Part. Each question carries 5 marks. (2×5=10)

21. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

22. Derive the formula for $\int_0^{\pi/2} \sin^p x \cos^q x \, dx$ where p and q are positive integers.

23. Show that the vectors $[2, 3, -1]$, $[1, -1, -2]$ and $[3, 1, 3]$ form a linearly independent set in \mathbb{R}^3 .

24. Diagonalize the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & -1 \\ -1 & -1 & 0 \end{bmatrix}$.

