K24U 0058

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/ Supplementary/Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Give an example of a step function defined on [1, 4].
2. Write norm of the partition P = (0, 5, 7, 9, 10) of [0, 10].
3. State additivity theorem.
4. Define Gamma function.
5. Define ε- neighborhood of a point x_0 in a metric space (S, d).

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. State non-uniform continuity criteria for a function $f : A \to \mathbb{R}$.
7. Using an example, show that product of monotonic increasing functions need not be increasing.
8. Let $f(x) = x^2$, $x \in [0,5]$. Calculate Riemann sum with respect to the partition P = (0, 1, 3, 5), take tags at the left end point of the subintervals.
9. Show that value of the integral of a Riemann integrable function is unique.

P.T.O.

10. If f is a Riemann integrable function and $k \in \mathbb{R}$, show that kf is Riemann integrable and $\int_a^b kf = k\int_a^b f$.

11. Evaluate $\int_1^{\infty} \frac{1}{x} dx$.

12. Show that B(m, n) = B(n, m).

13. Compute $\Gamma(-1/2)$.

14. Find pointwise limit of the sequence of functions (x^n) for $x \in [0,1]$.

15. Define a metric d on a set S.

16. State Cauchy criterion for convergence for sequence of functions.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Define uniformly continuous function. Show that $f(x) = x^2$ is not uniformly continuous on $[0, \infty)$.

18. Show that Riemann integrable functions defined on [a, b] are bounded on [a, b].

19. Show that if f, $g \in R[a,b]$, then $f + g \in R[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

20. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

21. From the definition of beta function, derive $B(m, n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$.

22. Derive $\Gamma(n) = \int_0^{\infty} [\log(1/t)]^{n-1} dt$.

23. Show that a sequence of bounded functions (f_n) defined on a set A converges uniformly on A to a function f if and only if $\| f_n - f \| \to 0$.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(6×2=12)**

24. a) Define a Lipschitz function. Show that Lipschitz functions are uniformly continuous.

    b) Show that not every uniformly continuous function is a Lipschitz function.

25. State and prove Fundamental theorem of calculus (1st form).

26. Show that $B(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{(m+n)}$.

27. a) Using an example, show that pointwise limit of a sequence of continuous functions need not be continuous.

    b) Given that (f_n) is a sequence of continuous functions defined on a set A such that (f_n) converges uniformly to a function f defined on A. Prove that f is continuous on A.