K26U 0198

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, April 2026
(2020 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the equation of the line in space through the points (3, –2, 4) and (–5, 7, 1).
2. If f and g are polynomials of degree n, then f + g is a polynomial of degree n. State true or false.
3. Define linearly dependent vectors.
4. State dimension theorem.
5. Find the eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Let S = {0, 1} and F = R. In F(S, R), show that f = g, where f(t) = 2t + 1, $g(t) = 1 + 4t - 2t^2$.
7. How many matrices are there in the vector space $M_{2\times2}(Z_2)$?
8. Prove that $(A^t)^t = A$ for each $A \in M_{m\times n}(F)$.

P.T.O.

9. Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly independent, prove that S_1 is linearly independent.

10. Show that the set $\{1, x, \ldots, x^n\}$ is linearly independent in $P_n(F)$.

11. Let V be a vector space and $\beta = \{u_1, u_2, \ldots, u_n\}$ be a subset of V. Then β is a basis for V if and only if each $v \in V$ can be uniquely expressed as a linear combination of vectors of β.

12. Show that no skew symmetric matrix can be f rank 1.

13. Write elementary column transformations of a matrix.

14. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & 5 \\ 1 & 2 & 4 \end{bmatrix}$.

15. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

16. Find the sum and the product of the eigenvalues of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **(8×2=16)**

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Let V be a vector space and W a subset of V. Prove that W is a subspace of V if and only if the following three conditions hold for the operations defined in V.
   a) $0 \in W$.
   b) $x + y \in W$ whenever $x \in W$ and $y \in W$.
   c) $cx \in W$ whenever $c \in F$ and $x \in W$.

18. Prove that $W_1 = \{ (a_1, a_2, \ldots, a_n) \in F^n : a_1 + a_2 + \ldots + a_n = 0 \}$ is a subspace of F^n.

19. Let V be vector space having a finite basis. Prove that every basis for V contains the same number of vectors.

20. Prove that $\left\{\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}, \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}\right\}$ forms a basis for $M_{2\times2}(R)$.

21. Let $T : R^3 \to R^2$ be defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Prove that T is a linear transformation.

22. Let $T : P_3(R) \to P_2(R)$ be the linear transformation defined by $T(f(x)) = f'(x)$. Let β and γ be the standard ordered bases for $P_3(R)$ and $P_2(R)$ respectively. Compute $[T]_\beta^\gamma$.

23. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}$. **(4×4=16)**

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. Solve :

$3x_1 - 7x_2 + 4x_3 = 10$
$x_1 - 2x_2 + x_3 = 3$
$2x_1 - x_2 - 2x_3 = 6.$

25. State and prove replacement theorem.

26. Let V and W be vector spaces over F and suppose that $\{v_1, v_2, \ldots, v_n\}$ is a basis for V. For $w_1, w_2, \ldots, w_n$ in W, prove that there exists exactly one linear transformation $T : V \to W$ such that $T(v_i) = w_i$ for $i = 1, 2, \ldots, n$.

27. Find the characteristic equation of the matrix $A = \begin{bmatrix}2 & 1 & 1\\0 & 1 & 0\\1 & 1 & 2\end{bmatrix}$ and hence compute A^{-1}. **(2×6=12)**