K25U 3077

Reg. No. : .....

Name : .....

III Semester B.Sc. Degree (CBCSS – OBE – Supplementary/Improvement) Examination, November 2025

(2023 Admission)

COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS FOR BSC ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING

3C03MAT-AIML : Differential Equations and Fourier Series

Time : 3 Hours Max. Marks : 40

PART – A

(Short Answer)

Answer **all** questions from this Part. **Each** question carries **1** mark. **(6×1=6)**

1. Define degree of a differential equation.
2. Define Wronskian.
3. State the order of the ODE $x^2y'' + \pi y^3 = 0$.
4. Verify that $y = 5e^{-2x} + 2x^2 + 2x + 1$ is a solution of $y' + 2y = 4(x + 1)^2$.
5. Solve $y' = ky$.
6. Define an even function in the context of Fourier series.

PART – B

(Short Essay)

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

7. Solve $y' + (x + 2)y^2 = 0$.
8. Find the integrating factor of $ydx - xdy = 0$.
9. Solve $\frac{dy}{dx} = xy + x$.
10. Find the Wronskian of e^{2x} and e^{-2x}.

P.T.O.

11. Find the general solution of $\frac{d^2y}{dx^2} - 4y = 0$.

12. Find a differential equation whose solution is cos3x.

13. Show that sum of two odd function is odd.

14. Write down the Euler formulae for calculating the Fourier coefficients of functions f(x) of period 2π.

PART – C
(Essay)

Answer **any four** questions from this Part. **Each** question carries **3** marks. **(4×3=12)**

15. Solve the initial value problem $y' + y\tan x = \sin 2x$, $y(0) = 1$.

16. Solve $xy' + y = xy^{\frac{3}{2}}$, $y(1) = 4$.

17. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = e^{2x}$.

18. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos 5x$.

19. Find the Fourier cosine transform of f(x) = 1 if $0 < x < a$ and f(x) = 0 if $x > a$.

20. Expand the function defined by f(x) = 0 if $-2 < x < 0$ and f(x) = x if $0 \leq x < 2$ as a Fourier series on $[-2, 2]$.

PART – D
(Long Essay)

Answer **any two** questions from this Part. **Each** question carries **5** marks. **(2×5=10)**

21. Check the exactness and solve $(2xy^2 + y)\,dx + (2y^3 - x)\,dy = 0$.

22. Solve the initial value problem $\left(y + \sqrt{x^2 + y^2}\right)dx - xdy = 0$, $y(1) = 0$.

23. Find the general solution of $y'' + 4y' + 4y = e^{-x}\cos x$.

24. Find a Fourier series that represents f(x) = |x| in $|-\pi, \pi]$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}$.