K26U 0196

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/ Supplementary/Improvement) Examination, April 2026
(2020 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
6B11MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

Answer **any four** questions. **Each** question carries **1** mark **each**.

1. Let $w = P(z) = z^2 + 3z$. Find the real part of w.
2. Find the exponential form of $\sqrt{i}$.
3. Show that $f(z) = e^z$ is periodic with period $2\pi i$.
4. Evaluate $\int_0^{-1+i} z^2 dz$.
5. Give an example of a simply connected domain. **(4×1=4)**

Answer **any eight** questions from the following. **Each** question carries **2** marks.

6. Prove that $P(z) = \overline{z}$ is not differentiable.
7. Show that a function of the form $f(z) = c_0 + c_1 z + ... + c_n z^n$ where $c_0, ..., c_n$ are complex constants is analytic in the entire complex plane.
8. Find the principal value of $\ln(-1)$.
9. Show that $\oint_C \frac{dz}{z} = 2\pi i$, where C is a circle of radius 1 and center 0, oriented counter clockwise.
10. Does the series $\sum \left(\frac{3i}{n}\right)$ convergent ? Justify your answer.
11. Find the Maclaurin series of $f(z) = \frac{1}{1-z}$.
12. Show that the zeros of a non-zero analytic function are isolated.
13. Evaluate $\lim_{z \to 1} \frac{z^4 - 1}{z^2 + 1}$.

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14. Show that the function $f(z) = z^2$ is continuous at $z = 0$.

15. Define the hyperbolic cosine and hyperbolic sine functions.

16. Prove or disprove : Every convergent series is absolutely convergent. **(8×2=16)**

Answer **any four** questions. **Each** question carries **4** marks **each**.

17. Determine a and b such that the function $u = e^{ax}\cos(by)$ is harmonic.

18. If $f(z) = u(x, y) + i\, v(x, y)$ is analytic in a domain D, then prove that both u and v satisfy Laplace's equation.

19. Find an upper bound for the absolute value of the integral $\int_C \frac{z^2}{z+2}\,dz$, where C is the circle : $|z| = 1.5$.

20. State and prove Liouville's theorem.

21. State and prove Cauchy's convergence principle for series.

22. Show that the power series $\sum_{n=0}^{\infty} n!z^n$ diverges for every $z \neq 0$.

23. Using Cauchy's Integral formula, integrate $f(z) = \frac{1}{z^3 - z^4}$ around the circle $C : |z| = \frac{1}{2}$ counter clockwise. **(4×4=16)**

Answer **any two** questions. **Each** question carries **6** marks **each**.

24. Prove the following :
    i) $|\cos z|^2 = \cos^2 x + \sinh^2 y$
    ii) $|\sin z|^2 = \sin^2 x + \sinh^2 y$.

25. Evaluate $\int_C \mathrm{Re}\ z^2 dz$, where C is the boundary of the square with vertices 0, i, 1+ i, 1 taken clockwisely.

26. i) Prove the following : If a series $\sum z_n$ converges, then $\lim_{n\to\infty} z_n = 0$.

    ii) Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(1+i)^n}{(2n)!}$.

27. State and prove the Residue theorem. **(2×6=12)**