M 7898

Reg. No. : .....

Name : .....

I Semester B.Sc. Degree (CCSS – Regular) Examination, November 2014
(2014 Admn.)
CORE COURSE IN MATHEMATICS
1B01 MAT : Differential Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

1. **All** the first **4** questions are **compulsory**. They carry **1** mark **each**.

   1) Find $\lim_{x \to -3} (5 - x)^{4/3}$.

   2) Rewrite the expression in term of exponentials : cosh 5x + sinh 5x.

   3) Define asymptote of a curve.

   4) Find $\lim_{(x,y) \to (0,1)} \frac{x - xy + 3}{x^2y + 5xy - y^3}$. **(4×1=4)**

SECTION – B

Answer **any 8** questions from **5** to **14**. They carry **two** marks **each**.

5. If $2 - x^2 \leq g(x) \leq 2 \cos x$ for all x, find $\lim_{x \to 0} g(x)$.

6. Find $\frac{d}{dt}(\tanh \sqrt{1 + t^2})$.

7. If $y = e^{ax} \sin bx$, prove that

   $y_2 - 2ay_1 + (a^2 + b^2)\, y = 0$.

P.T.O.

8. Find the Cartesian coordinate of the point $(2, \pi/3)$.

9. Graph the set of points whose polar coordinates satisfy the inequality $0 \le r \le 2$.

10. Verify Rolle's Theorem for $f(x) = (x+2)^3 (x-3)^4$ in $(-2, 3)$.

11. For the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\frac{ds}{dx}$.

12. Find $\lim_{x \to 0} \frac{1-\cos x}{x+x^2}$.

13. Find the domain and range of the function $w = \sqrt{y - x^2}$.

14. Verify Euler's theorem on homogeneous functions $z = 10x^2 + 7xy + 5y^2$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from **15** to **20**. They carry **4** marks.

15. If $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

16. Prove that $\lim_{x \to 4}(9 - x) = 5$.

17. Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

18. Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $(0, 2)$?

19. Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$. What is the derivatives value at $t = 0$.

20. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, show by Euler's theorem that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from **21** to **24**. They carry **6** marks **each**.

21. If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$. Hence find $(y_n)_0$. **6**

22. Find the polar equation for the circle $x^2 + 2x + y^2 = 0$. Sketch the circle in the coordinate plane and label it with both its Cartesian and polar equations. **6**

23. Use Taylor's theorem to prove that $\tan^{-1}(x + h) = \tan^{-1} x + (h \sin z)\, \frac{\sin z}{1}$

$-(h \sin z)^2 \frac{\sin 2z}{s} + \ldots\ldots$ where $z = \cot^{-1} x$. **6**

24. Find the linearization L(x, y) of the function at the given point :

a) $f(x, y) = e^x \cos y$ at (0, 0)

b) $f(x, y) = x^3 y^4$ at (1, 1). **6**

(2×6=12)