M 7937

Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, May 2010
MATHEMATICS (Core)
Course No. 2 : 2B02 MAT : Foundation of Higher Mathematics

Time: 3 Hours Maximum Weightage : 30

Fill the blanks :

1. a) For $|x| < 1$, $1 - x + x^2 - x^3 + ... =$ _____

   b) Sum of the series $2\left[1+\frac{1}{3!}+\frac{1}{5!}+.....\right] =$ _____

   c) Coefficient of x^n in the expansion of $2xe^{2x}$ is _____

   d) n^{th} term of the series $\frac{9}{1!}+\frac{16}{2!}+\frac{27}{3!}+\frac{42}{4!}+$ _____ **(Weightage 1)**

2. a) $f(x) = x^2$, $9(x) = x + 3$ then $(g \circ f)(2) =$ _____

   b) The set $[a] = \{x \in A : x \sim a\}$ where $\sim$ is an equivalence relation on set A is called _____

   c) A partition of set $X = \{1, 2, 3, 4\}$ is _____

   d) Domain of $f(x) = \sqrt{25 - x^2}$ is _____ **(Weightage 1)**

Answer **any five** from the following : **(Weightage 1 each)**

3. Sum the series $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+$ _____

4. Sum the series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+$ _____

P.T.O.

5. Suppose A = {a, b, c} and B = (1, 2}. Then find the number of functions from A to B which are onto.

6. R = {(1, 2), (1, 3), (3, 1), (3, 3), (2, 3)} is a relation on A = {1, 2, 3} find R o R.

7. Sketch the product set [–3, 2) × (–2, 2] in the plane R^2.

8. Let $f : R \to R$ be defined by $f(x) = \frac{2}{3}n + \frac{4}{5}$ find the formula for $f^{-1}(x)$.

9. Define partial order on a set S.

10. Suppose the set P = {1, 2, 3....} of positive integers is ordered by divisibility. Insert the Correct Symbol <, ≥ or || between each pair of numbers.

    a) 2 – 8 b) 18 – 24

    c) 9 – 3 d) 5 – 15 **(Weightage 5×1=5)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Let A be a set of non-zero integers and let ≈ be a relation on A×A defined as follows (a, b) ≈ (c, d) whenever ad = bc. Prove that ≈ is an equivalence relation.

12. Let A = {1, 2, 3, 4, 6}. Let R be a relation on A defined by x divides y.

    a) Write R as a set of ordered pairs

    b) Draw its directed graph

    c) Find the inverse relation R^{-1} of R

    d) Can R^{-1} be described in words.

13. Sketch the relation $3x^2 + 4y^2 \leq 12$. Find the domain of this relation.

14. Let $f : A \to B$ and $g : B \to C$. Then if $g \circ f$ is one-to-one prove that f is one-to-one.

15. Define a partial order relation on a set S. When S is linearly ordered. Give an example of a set with a partial order which is not linearly ordered.

16. Define a lattice, sub-lattice and isomorphic lattices.

17. Consider the relation R = {(1,1), (1, 3), (2, 4), (3, 1), (3, 2)} on A = {1, 2, 3, 4} find :
    a) Reflexive closure of R
    b) Symmetric closure of R
    c) Transitive closure of R.

18. Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$ whose roots are in arithmetical progression.

19. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the real value of $\Sigma\alpha^2\beta$.

20. Find the equation whose roots are the roots of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. Show that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

22. Sum to infinities the series $\frac{1}{1.2.3} + \frac{1}{5.6.7} + \frac{1}{9.10.11} + \ldots\ldots$

23. Let L be a Lattice. Then prove that :
    i) $a \wedge b = a$ if and only if $a \vee b = b$
    ii) The relation $a \leq b$ defined by $a \wedge b = a$ is a partial order relation on L.

24. Solve the equation $x^3 - 21x - 344 = 0$ by Cardan's method.

25. If the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ be α, β, γ find the equation whose roots are $\alpha^2 + \beta^2, \beta^2 + \gamma^2, \gamma^2 + \alpha^2$. **(Weightage 3×3=9)**
