**M 8004**



**Reg. No. : .................................**

**Name : .....................................**

## Second Semester B.Sc. Degree Examination, May 2010
## STATISTICS (Complementary)
## Course No. 2 : 2C02 STA : Probability Theory and Random Variables

Time : 3 Hours　　　　　　　　　　　　　　　　Total Weightage : 30

***Instruction** : Use of calculators and statistical tables **permitted**.*

PART – A

Answer **any 10** questions :　　　　　　　　　　　　　**(Weightage 1 each)**

1. Give the classical definition of Probability.
2. What is statistical regularity ?
3. A coin is tossed continuously, till head appears for the first time. Write down the sample space.
4. What is the probability that a leap year selected at random will contain 53 Sundays ?
5. State the addition rule for two events.
6. If A, B, C are three events defined on the same sample space, state the condition for their total independence.
7. Explain conditional probability giving an example.
8. A random variable X has the following density function :

   $f(x) = \frac{k}{1+x^2}$ ; $-\infty < x < \infty$. Find the value of k.
9. State the important properties of distribution function.
10. The joint density function of X and Y is

    $$f(x, y) = \begin{cases} 8xy & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

    Find the marginal density function of Y.

**P.T.O.**

11. Two random variables X and Y have the following joint distribution :

| X \ Y | 1 | 2 |
|---|---|---|
| −1 | $\frac{1}{30}$ | $\frac{2}{30}$ |
| 1 | $\frac{3}{10}$ | $\frac{3}{5}$ |

Examine the independence of X and Y.

PART – B

Answer **any 6** questions : **(Weightage 2 each)**

12. Explain the aniomatic definition of probability.

13. A and B are two events defined on the same sample space such that $P(A)=p_1, P(B)=p_2, P(A\cap B)=p_3$.

Find : 1) $P(A'\cap B')$ 2) $P(A'\cup B')$ 3) $P(A'\cup B)$

in terms of $p_1$, $p_2$, $p_3$.

14. Two balanced dice are thrown simultaneously. Find the probability that the sum of the numbers shown by the dice is

1) Greater than 8

2) Neither 7 nor 11.

15. A bag contains 12 white, 6 red and 7 black balls. Three balls are drawn at random without replacement. What is the probability that two of them are white and the other is black ?

16. State and prove Bayes theorem.

17. Define pairwise independence of three events. Show by an example that pairwise independence does not mean total independence.

18. The distribution function of a random variable X is given as follows :

$$F(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4} & \text{if } -1 \le x < 1 \\ \frac{2}{3} & \text{if } 1 \le x < 5 \\ \frac{7}{8} & \text{if } 5 \le x < 10 \\ 1 & \text{if } x \ge 10 \end{cases}$$

Find the probability density (mass) function.

19. A continuous random variable X has the following density function :

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the density function of $-2 \log X$.

20. The random variables X and Y have the joint mass function.

$$f(x, y) = \frac{1}{27}(x + 2y) \, ; \quad \begin{matrix} x = 0, 1, 2 \\ y = 0, 1, 2 \end{matrix}$$

Find the marginal distributions.

Also find the conditional distribution of

1) Y for X = 0 and 2) X for Y = 2.

PART – C

Answer **any two** questions : **(Weightage 4 each)**

21. A factory produces pipes in three plants. The daily production figures are as follows.

Plant I : 3000 numbers

Plant II : 2500 numbers

Plant III : 4500 numbers

Past experience shows that 1 percent of the pipes produced by Plant I are defective. The corresponding percentages of defectives of Plant II and III are 1.2 percent and 2 percent respectively. A pipe is drawn at random from a day's production and is found to be defective. What is the probability that it comes from

1) Plant I

2) Plant II

3) Plant III ?

22. A random variable X has the following density function :

$f(x) = kx(1 - x)$ if $0 \leq x \leq 1$

1) Find the value of k.

2) Determine the number b such that $P\{X < b\} = P\{X > b\}$.

3) Also find the distribution function of X.

23. A random variable X has the following probability distribution :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Value of X** : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **Probability** : | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

Find :

1) the value of k,

2) $P\{X < 6\}$

3) $P\{0 < X < 5\}$

4) the minimum value of 'a' such that $P\{X \leq a\} > \frac{1}{2}$.

24. Define joint probability density function, marginal and conditional probability density functions. Two random variables X and Y have the following joint density functions :

$$f(x, y) = \frac{1}{4}(1 + xy); \quad \begin{array}{l} -1 < x < 1 \\ -1 < y < 1 \end{array}$$

Examine the independence of 1) X and Y 2) $X^2$ and $Y^2$.

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