M 3817

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II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS-Reg./Supple./Improv.) Examination, May 2013
CORE COURSE IN MATHEMATICS
2B02 MAT : Foundation of Higher Mathematics

Time: 3 Hours Max. Weightage: 30

1. Fill the blanks :

   a) The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\ldots=$ ________

   b) For $x \in \mathbb{R}$ the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots=\ln e^{(1+x)}$ when x belongs to the interval ____________

   c) n-th term of the series $\frac{9}{1!}+\frac{16}{2!}+\frac{27}{3!}+\frac{42}{4!}+\ldots$ is ______________

   d) Coefficient of x^n in the expansion of $2xe^{2x}$ is ____________ **(Weightage 1)**

2. a) The number of relations from set $A=\{a, b, c\}$ to set $B=\{1, 2\}$ is ____________

   b) The set $[a]=\{x \in A : x \sim a\}$ where ~ is an equivalence relation on Set A is called ____________

   c) Let $f : R \to R$ is defined by $f(x)=x^2+2x$. Then $(f \circ f)(3)=$ __________

   d) Domain of the real valued functions $f(x)=\sqrt{x^2-25}$ is __________ **(Weightage 1)**

Answer **any five** from the following (Weightage **1 each**) :

3. Sum the series $1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

4. Sum the series $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+\ldots$

P.T.O.

5. Find the matrix of the relation R = {(1, y), (1, z), (2, x), (2, z), (3, y), (4, x), (4, y)} defined from A = {1, 2, 3, 4} to B = {x, y, z}.

6. Sketch the product set $[-3, 2) \times (-2, 2]$ in the plane R^2.

7. Consider the formula $f(x) = x^2, x \in R$. Find the largest interval D such that $f : D \to R$ is a one-to-one function.

8. Suppose A = {a, b, c} and B = {1, 2}. Then find the number of on to functions from A to B.

9. Define a partial order on a set S.

10. Prove that $a \wedge a = a$ for every a where a is an element of a Lattice L.

(Weightage 5×1=5)

Answer **any seven** from the following (Weightage **2 each**).

11. Let A = {1, 2, 3, 4, 6}. Let R be a relation on A defined by x divides y.
    a) Write R as a set of ordered pairs
    b) Draw its directed graph
    c) Find the inverse relation R^{-1} of R
    d) Can R^{-1} be described in words.

12. Let A be a set of nonzero integers and let $\approx$ be a relation on $A \times A$ defined as follows

    $(a, b) \approx (c, d)$ whenever ad = bc. Prove that $\approx$ is an equivalence relation.

13. Draw the graph of the function $f(x) = \begin{cases} -x, & x \le -1 \\ 1, & -1 < x < 1. \\ x^2, & x \ge 1 \end{cases}$

14. Let $f : A \to B$ and $g : B \to C$. Then if $g \circ f$ is one-to-one prove that f is one-to-one.

15. Consider the relation R = {(1, 1), (1, 3), (2, 4), (3, 1), (3, 2), (4, 3)} on A = {1, 2, 3, 4}. Find :
    a) Reflexive closure of R
    b) Symmetric closure of R
    c) Transitive closure of R.

16. Define a lattice, sublattice and isomorphic lattices.

17. Consider the ordered set A as pictured in fig. 1.

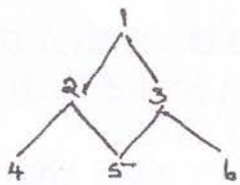

Fig. 1

a) Find all maximal elements and minimal elements.

b) Does A have a first element and last element.

18. If –4 is a root of $2x^3 + 6x^2 + 7x + 60 = 0$ find the other roots.

19. Find the equation whose roots are the roots of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4.

20. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + v = 0$ find the value of $\sum \frac{1}{\alpha^2}$.

(Weightage 7×2=14)

Answer **any three** from the following (Weightage **3 each**) :

21. Show that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

22. Sum the series $\frac{1}{1.2.3} + \frac{5}{3.4.5} + \frac{9}{5.6.7} + \cdots$

23. Let L be a Lattice. Then prove that

    i) $a \wedge b = a$ if and only if $a \vee b = b$

    ii) The relation $a \le b$ defined by $a \wedge b = a$

    Is a partial order relation on L.

24. Solve $x^3 - 18x - 35 = 0$, by Cardan's method.

25. If the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ be α, β, γ find the equation whose roots are $\alpha^2 + \beta^2, \beta^2 + \gamma^2, \gamma^2 + \alpha^2$.

(Weightage 3×3=9)
