M 3831

Reg. No. : ....

Name : ....

II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2013

COMPLEMENTARY COURSE IN STATISTICS

(For Maths/Comp-Sci. Core)

2C02 STA : Probability Theory and Random Variables

Time : 3 Hours Total Weightage : 30

***Instruction** : **Use** of calculators and statistical tables **permitted**.*

PART – A

Answer **any 10** questions : Weightage **1** each :

1. A coin is tossed from times. Write down the sample space.
2. Give the empirical definition of probability.
3. Using the axisms of probability, show that $P(\overline{A}) = 1 - P(A)$.
4. If A and B are any two events defined on the same simple space, prove that $P(A \cup B) \leq P(A) + P(B)$.
5. Distinguish between mutually exclusive events and independent events.
6. If A and B are independent events, show that $\overline{A}$ and $\overline{B}$ are also independent.
7. If B_1, B_2, B_3 is a partitioning of a sample space s and A is any other event defined on s, show that $P(A) = \sum_{i=1}^{3} P(Bi)\, P(A \mid Bi)$.
8. A random variable X has the following density function

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the distribution function of X.

P.T.O.

9. A discrete random variable X has the following probability distribution

Value of X :	-1	0	1	2
Probability :	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$

Find the distribution of X^2.

10. Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal density of X.

11. The joint probability mass function of X and Y is given as follows :

$$f(x, y) = \frac{2x + y}{27}; \quad \begin{matrix} x = 0, 1, 2 \\ y = 0, 1, 2 \end{matrix}$$

Find $p\{y = 0\}$. **(10×1=10)**

PART – B

Answer **any six** questions : Weightage **2** each :

12. Define σ-field. Show that probability measure is defined over a σ-field.

13. If $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find the probabilities of
    1) At least one of the events occur
    2) Exactly one of the events occur
    3) None of the events occur.

14. Four cards are drawn at random from a pack of 52 cards. Find the probability that
    1) They are a king, a queen, a jack and an ace
    2) Two are kings and two are queens
    3) Two are black and two are red
    4) Two are hearts and two are diamonds.

15. State and prove Bayes theorem.

16. Distinguish between pairwise independence and total independence. Using an example show that pairwise independence doesnot imply intal independence.

17. A random variable X has the following density function

$$f(x) = \begin{cases} kx & \text{if } 0 \le x \le 1 \\ k & \text{if } 1 \le x \le 2 \\ -kx + 3k & \text{if } 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find (1) the value of k (2) $P\{X < 1.5\}$

18. Let X be a continuous random variable with pdf

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \; ; -\infty < x < \infty.$$

Find the pdf of X^2.

19. Two random variables X and Y have the joint density function

$$f(x, y) = \begin{cases} 4(1-x-y) & ; x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal densities of X and Y. Hence examine if X and Y are independent.

20. The joint probability mass function of X and Y is

$$f(x, y) = \frac{x+y}{21} ; \begin{matrix} x = 1, 2, 3 \\ y = 1, 2 \end{matrix}$$

Find the marginal distributions of X and Y.
Also find the conditional distribution of X for Y =1. **(6×2=12)**

PART – C

Answer **any two** questions : Weightage **4** each :

21. State and prove the addition rule for two events. The contents of three urns are as follows :

Urn I : 1 white, 2 black and 3 red balls

Urn II : 2 white, 1 black and 1 red balls

Urn III : 4 white, 5 black and 3 red balls

one urn is chosen at random and two balls are drawn. They happen to be white and red. What are the probabilities that they come from urn 1, urn II, urn III ?

22. A continuous random variable X has the following density function :

$$f(x) = \begin{cases} A\ e^{-x/5} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

1) Find the value of A

2) Show that for any two positive numbers s and t

$P\{X > s + t \mid X > s\} = P\{X > t\}$.

3) Also find the distribution function of X.

23. A random variable X has the following probability distribution.

Value of X :	0	1	2	3	4	5	6	7	8
Probability :	k	3k	5k	7k	9k	11k	13k	15k	17k

1) Determine the value of k.

2) Find $P\{X < 3\}$, $P\{X \geq 3\}$, $P\{0 < X < 5\}$

3) What is the smallest value of x for which $P\{X \leq x\} > 0.5$?

4) Find the distribution function of X.

24. Define joint probability density function, marginal and conditional probability density functions.

A two dimensional random variable (X, Y) has the following joint density function.

$$f(x, y) = \frac{1}{4}(1 + xy) \quad \begin{matrix} \text{if } -1 < x < 1 \\ -1 < y < 1 \end{matrix}$$

Examine the independence of

1) X and Y

2) X^2 and Y^2. **(2×4=8)**

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