M 8869

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – 2014 Admn. – Regular)
Examination, May 2015
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. State the mean value theorem for definite integrals.

2. Evaluate : $\int_0^{\infty} x^3 e^{-x}\, dx$.

3. Fill in the blanks : The equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} + 2z = 0$ represents a surface known as ____________

4. Evaluate : $\int_{-1}^{1} \int_{0}^{2} (1 - 6x^2 y)\, dx dy$.   **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. They carry **2** marks **each**.

5. Show that if f is continuous on [a, b], a $\neq$ b and if $\int_a^b f(x)\, dx = 0$ then f(x) = 0 at least once in [a, b].

6. Define the Riemann sum of a continuous function f defined on the interval [a, b].

7. Evaluate : $\int_0^1 \sinh^2 x dx$.

P.T.O.

8. Test for convergence : $\int_1^\infty \frac{x\,dx}{3x^4+5x^2+1}$.

9. Prove that $\Gamma(n+1) = n\,\Gamma(n), n > 0$.

10. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

11. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

12. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$.

13. Evaluate $\int_0^1 \int_0^x (3 - x - y)\, dy\, dx$.

14. Find the area enclosed by the lemniscate $\gamma^2 = 4\cos 2\theta$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. They carry **4** marks **each**.

15. Find $\int x \sin^{-1} x\, dx$.

16. Examine for convergence $\int_0^\infty \frac{1-\cos x}{x^2} dx$.

17. The line segments $x = 1 - y;\ 0 \le y \le 1$ is revolved about the y-axis to generate the cone. Find its lateral surface area.

18. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = 3/2$ to generate a solid. Find the volume of the solid.

19. Find the volume of the region enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

20. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. They carry **6** marks each.

21. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x; -1 \leq x \leq 2$.

22. Prove that $B(m, n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$; $m, n > 0$.

23. Find the area inside the smaller loop of the limacon $\gamma = 2\cos\theta + 1$.

24. Find the volume of the upper region D cut from the solid sphere $p \leq 1$ by the cone $\phi = \pi/3$. **(2×6=12)**
