K19U 0579

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (CBCSS – Reg.) Examination, April 2019
(2017 Admission Only)
Core Course in Mathematics
4B 04 MAT : ELEMENTS OF MATHEMATICS – II

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find the number of relations from A = {a, b, c} to B = {1, 2}.
2. Find 29(mod 6).
3. Find the rank of the unit matrix of order n.
4. State true of false : Elementary transformations change the rank of a matrix.

SECTION – B

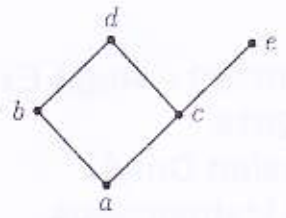
Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find the domain of $f(x) = \sqrt{25 - x^2}$.
6. Let the function $f : R \to R$ be defined as follows $f(x) = \begin{cases} 3x - 1 & \text{if } x > 3 \\ x^2 - 2 & \text{if } -2 \le x \le 3 \\ 2x + 3 & \text{if } x < -2 \end{cases}$

   Find f(2) and f(4).
7. Let f and g be two functions defined by f(x) = 2x + 1 and $g(x) = x^2 - 2$. Find $f \circ g$ and $g \circ f$.
8. Let m be a positive integer and let D_m denote the set of divisors of m ordered by divisibility. Find D_{36} and draw the Hasse diagram of D_{36}.

P.T.O.

9. Let S = { a, b, c, d, e} be ordered as in figure shown below



a) Find all minimal and maximal elements of S.

b) List all chains with three or more elements.

10. Find the equation of pair of tangents from a given point P(h, k) to the parabola $y^2 = 4ax$.

11. Find the equation of the chord of an ellipse joining two points whose eccentric angles are θ and ϕ.

12. Let $P\left(ct_1, \frac{c}{t_1}\right)$ and $P\left(ct_2, \frac{c}{t_2}\right)$ be any two points on the hyperbola $xy = c^2$. Find the equation of the tangent at P.

13. In a rectangular hyperbola, prove that $SA \times SA' = a^2$, where S is one of the foci and A, A′ are the vertices of the hyperbola.

14. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks each.

15. Find all partitions of S = {a, b, c, d}.

16. Consider the functions $f : A \to B$ and $g : B \to C$. Prove the following

a) If f and g are one to one, then the composition function $g \circ f$ is one to one.

b) If f and g are onto functions, then the composition function $g \circ f$ is an onto function.

17. Consider the set Z of integers. Define aRb by $b = a^r$ for some positive integer r. Show that R is a partial order on Z.

18. Find the equation of the tangent at P(h, k) on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

19. Show that the foot of the perpendicular drawn from the focus on any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ lies on a circle.

20. Reduce the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to its normal form and hence find the rank of A.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Consider the set Z of integers. Define a relation ~ on Z by x ~ y if x – y is divisible by 5. Prove that ~ is an equivalence relation and find corresponding equivalence classes.

22. Let L be a finite distributive lattice. Then every a in L can be written uniquely as the join of irreduntant join-irreducible elements.

23. Prove that the orthocenter of a triangle formed by the tangents to the parabola lies on its directrix.

24. Compute the inverse of $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ using elementary row transformations.