K21P 4213

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)
Examination, October 2021
(2018 Admission Onwards)
MATHEMATICS
MAT1C05 : Differential Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks **each**.

1. Find a power series solution of the differential equation $y' = 2xy$.

2. Locate and classify the singular points of

   i) $x^2\left(x^2-1\right)^2 y'' - x(1-x)y' + 2y = 0$

   ii) $x^4 y'' + (\sin x)y = 0$.

3. State the generating function for the Legendre polynomial $P_n(x)$. Use it to prove that $P_{2n}(0) = (-1)^n \dfrac{1.3.\cdots(2n-1)}{2^n n!}$.

4. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\sin x$.

5. Obtain the normal form of Bessel equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.

6. Find the first three approximate solutions of the initial value problem $y' = y^2$, $y(0) = 1$ using Picard's method.

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks **each**.

UNIT – 1

7. a) Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x. 7

b) Solve the differential equation $2x^2y'' + x(2x+1)y' - y = 0$. 9

8. a) Find the indicial equation and its roots of the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$. 6

b) Find two independent Frobenius solutions of the equation $x^2y'' - x^2y' + (x^2 - 2)y = 0$. 10

9. Find the general solution of the Gauss Hypergeometric differential equation. 16

UNIT – 2

10. a) If $P_m(x)$ and $P_n(x)$ respectively are m^{th} and n^{th} Legendre polynomials, then prove that

$$\int_{-1}^{1} P_m(x)\,P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

10

b) Obtain the recursion formula $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$. 6

11. a) Solve the Bessel equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ to get the Bessel function of first kind of order p. 9

b) Prove that :

i) $\dfrac{d}{dx}J_0(x) = -J_1(x)$

ii) $\dfrac{d}{dx}xJ_1(x) = xJ_0(x)$. 7

12. a) Find the general solution of the system of homogeneous equations

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y$$

9

b) If W(t) is the Wronskian of two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system of equations

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

then prove that W(t) of solutions is either identically zero or nowhere zero on [a, b]. 7

UNIT – 3

13. a) If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$, then prove that the zeros of these functions are distinct and occur alternatively in the sense that $y_1(x)$ vanishes exactly once between any two successive zeros of $y_2(x)$ and conversely. 8

b) Let u(x) be any non-trivial solution of $u'' + q(x)u = 0$, where $q(x) > 0$ for all $x > 0$. If $\int_0^{\infty} q(x)dx = \infty$, then prove that u(x) has infinitely many zeros on the positive x-axis. 8

14. Let f(x, y) be a continuous function that satisfies Lipschitz condition $|f(x, y_1) - f(x, y_2)| \leq K|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty \leq y \leq \infty$. If (x_0, y_0) is any point of the strip, prove that the Initial Value Problem $y' = f(x, y)$, $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$. 16

15. a) Show that f(x, y) = xy satisfy Lipschitz condition on any rectangle $a \leq x \leq b$, $c \leq y \leq d$. Also prove that f(x, y) does not satisfy the Lipschitz condition in the entire plane. 8

b) Solve the Initial Value Problem using Picard's method of successive method of approximation.

$$\frac{dy}{dx} = z \qquad y(0) = 1$$

$$\frac{dz}{dx} = -y \qquad z(0) = 0.$$ 8