K24U 0395

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – Supplementary/One Time Mercy Chance) Examination, April 2024
(2014 to 2018 Admissions)
Core Course in Mathematics
6B13MAT : MATHEMATICAL ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. If $P = \{a = x_0, x_1, x_2, \ldots x_n = b\}$ is a partition of $[a, b]$, then the Riemann upper sum of a function $f : [a, b] \to R$, is

2. Evaluate $\lim(f_n(x))$ where $f(x) = \dfrac{nx}{(1+n^2x^2)}$ for $x \in R$, $n \in N$.

3. A topological space is said to be separable if it has

4. Let X be an arbitrary metric space and $A \subseteq X$. Then Int(A) = **(4×1=4)**

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. If $h(x) = x^2$ on [0, 1] and $P_n = \left\{0, \dfrac{1}{n}, \dfrac{2}{n}, \ldots \dfrac{n-1}{n}, 1\right\}$ then find $\lim\limits_{n\to\infty} (U(P_n, h) - L(P_n, h))$.

6. If $f \in R\,[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, then show that $\left|\int_a^b f\right| \leq M(b-a)$.

7. Give an example for a bounded nonintegrable function on [0, 1].

8. Discuss the convergence of sequence (x^n) for $x \in R$.

9. State Weierstrass M-Test.

10. Determine the radius of convergence of the power series $\sum\left(1+\dfrac{1}{n}\right)^{n^2} x^n$.

11. Let X be a non-empty set and define d by $d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$. Show that d is a metric on X.

P.T.O.

12. Prove that in a metric space X, the complement of a closed set is open.
13. Prove that $\overline{A}$ equals the intersection of all closed supersets of A.
14. Show that the intersection of two topologies on a non-empty set X is also a topology on X. **(8×2=16)**

SECTION – C

Answer **any four** questions. **Each** question carries 4 marks.

15. Show that if $f : [a, b] \to R$ is monotone on [a, b], then f is integrable on [a, b].
16. State and prove the Fundamental Theorem of Calculus (First Form).
17. If $\{f_n\}$ is a sequence of continuous functions on a set $A \subseteq R$ converging uniformly on A to a function $f : A \to R$, then f is continuous on A.
18. Show that a subset of a topological space is perfect if and only if it is closed and has no isolated points.
19. If X is a complete metric space and Y is a subspace of X, prove that Y is complete if and only if it is closed.
20. Let X be an infinite set. Show that $T = \{U \subseteq X : U = \phi \text{ or } X \setminus U \text{ is finite}\}$ is a topology on X. **(4×4=16)**

SECTION – D

Answer **any two** questions. **Each** question carries 6 marks.

21. State and prove Riemann's Criterion for integrability.
22. Let (f_n) be a sequence of bounded functions on $A \subseteq R$. Prove that this sequence converges uniformly on A to a bounded function f if and only if for each $\epsilon > 0$ there is number $H(\epsilon)$ in N such that for all $m, n \geq H(\epsilon)$, then $||f_m - f_n||\, A \leq \epsilon$.
23. Show that in a metric space X :
    a) any union of open sets is open and
    b) any finite intersection of open sets is open.
24. Let $f : X \to Y$ be a mapping of one topological space into another. Show that f is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y. **(2×6=12)**