K24U 0392

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – Supplementary/ One Time Mercy Chance) Examination, April 2024
(2014 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark each. **(4×1=4)**

1. Define Nullity of a linear transformation.

2. State Cayley Hamilton theorem.

3. Solve the system of equations : $x + 2y + 3z = 14$, $y + 5z = 17$, $z = 3$.

4. Find the product of the characteristic roots of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

SECTION – B

Answer **any 8** questions. **Each** question carries **2** marks. **(8×2=16)**

5. Find the equation of the line through the points (3, –2, 4) and (–5, 7, 1) in space.

6. Give a basis for $M_{2\times2}(R)$, where R is the set of real numbers.

7. Show that the transformation $T : R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ is linear.

8. Show that the linear transformation $T : P_2(R) \to P_2(R)$ defined by $T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$ is one-to-one.

9. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

10. Show that the equations are not consistent

$2x + 6y + 11 = 0,\ 6x + 20y - 6z + 3 = 0,\ 6y - 18z + 1 = 0.$

11. Show that if $\lambda_1, \lambda_2, \ldots \lambda_n$ are n eigen values of a square matrix A of order n, then the eigen values of the matrix A^2 are $\lambda_1^2, \lambda_2^2, \ldots \lambda_n^2$.

12. Show that the characteristic roots of a Hermitian matrix are all real.

13. Test for diagonalizability of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

14. Let T be a linear operator on an n-dimensional vector space V. If T has n distinct eigen values, then show that T is diagonalizable.

SECTION – C

Answer **any 4** questions. Each question carries **4** marks. **(4×4=16)**

15. Define a vector space and give an example.

16. Determine whether the vector (–2, 0, 3) in R^3 can be written as a linear combination of the vectors (1, 3, 0) and (2, 4, –1).

17. $T : R^2 \to R^3$ is a linear transformation defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$. Find $[T]_\beta^\gamma$, where β and γ are the standard ordered bases for R^2 and R^3 respectively.

18. Find the characteristic roots and the corresponding characteristic vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

19. Solve the system of equations :

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0.$

20. Solve the system by Gauss-Jordan method

$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16.$

SECTION – D

Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

21. Show that $S = \{(2, -3, 5), (8, -12, 20), (1, 0, -2), (0, 2, -1), (7, 2, 0)\}$ generates R^3.

22. Let $T : R^2 \to R^3$ and $U : R^2 \to R^3$ be linear transformations respectively defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$. Verify that $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$, where β and γ are the standard ordered bases of R^2 and R^3 respectively.

23. Investigate for what values of λ and μ, the simultaneous equation $x + 2y + z = 8$, $2x + y + 3z = 13$, $3x + 4y - \lambda z = \mu$ have no solution.

24. Find the inverse of the matrix using Gauss elimination method $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.