K24U 0730

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark.

1. State Division algorithm.
2. Write the positive divisors of 30.
3. What do you mean by least common multiple of two integers ?
4. State Fundamental theorem of Arithmetic.
5. State Wilson's theorem. **(4×1=4)**

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks.

6. Let a, b and c be three integers. If a|b and b|c, then prove that a|c.
7. If k > 0, then prove that gcd(ka, kb) = k gcd(a, b).
8. Using division algorithm, find the gcd of 143 and 227.
9. If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then prove that $a \equiv c \pmod{n}$.

P.T.O.

10. Prove that 41 divides $2^{20} - 1$.

11. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$.

12. Let f be continuous on the symmetric interval [– a, a]. If f is even, prove that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

13. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

14. What are the steps to find the volume of a solid using area of cross section ?

15. The base of a solid is the region bounded by the graphs of $y = 3x$, $y = 6$ and $x = 0$. The cross-sections perpendicular to the x- axis are rectangles of height 10. Find the volume of the solid.

16. The circle $x^2 + y^2 = a^2$ is rotated about the x-axis to generate a sphere. Find its volume. **(8×2=16)**

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks.

17. Show that the expression $\frac{a(a^2+2)}{3}$ is an integer for $a \geq 1$.

18. Find the remainder when the sum 1! + 2! + ... 100! is divisible by 12.

19. Using Euler's theorem, prove that for any integer a, $a^{37} \equiv a \pmod{1729}$.

20. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below the x-axis and the line $y = x - 2$ by integrating with respect to x.

21. Find the length of the curve $y = \frac{x^3}{12} + \frac{1}{x}, 1 \leq x \leq 4$.

22. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

23. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$. **(4×4=16)**

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks.

24. Let a and b two integers, not both of which are zero. Prove that there exist integers x and y such that gcd(a, b) = ax + by.

25. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{p}$.

26. Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$.

27. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis. **(2×6=12)**