K24U 0394

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – Supplementary/One Time Mercy Chance) Examination, April 2024
(2014 to 2018 Admissions)
Core Course in Mathematics
6B12MAT : COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If $z_1 = 8 + 3i$ and $z_2 = 9 - 2i$ then $\text{Im}(z_1 z_2) =$
2. Give an example for a function which has a simple pole at the point z = 0.
3. The residue of $f(z) = \frac{4}{1+z^2}$ at z = i is
4. Define removable singularity. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Evaluate $\int_C \text{Re}(z)dz$, from z = 0 to z = 1 + 2i along C, where C is the line segment joining the points (0, 0) and (1, 2).
6. Evaluate $\oint_C \frac{dz}{z-3i}$, where C is the circle $|z| = \pi$ in counter clockwise.
7. State and prove Liouville's theorem.
8. Define absolutely convergent and conditionally convergent of a series.

P.T.O.

9. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$ and write its circle of convergence.

10. a) State ratio test.

    b) Prove that the derived series of a power series has the same radius of convergence as the original series.

11. Evaluate the residue of $\frac{9z+i}{z(z^2+1)}$ at $z = i$.

12. Find the Laurent series of $f(z) = z^2 e^{\frac{1}{z}}$ with center $z = 0$.

13. Define isolated essential singularity and pole of order m. Give an example for a function which has isolated essential singularity.

14. State Laurent's Theorem. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions 15 to 20. These questions carry **4** marks **each.**

15. Verify that the $u(x, y) = x^3 - 3xy^2$ is harmonic in the whole complex plane and find a harmonic conjugate function $v(x, y)$ of $u(x, y)$.

16. a) Show that $\cosh z = \cosh x \cos y + i \sinh x \sin y$.

    b) Show that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

17. Expand $f(z) = \frac{1}{z(z-1)}$ in Laurent series valid for $0 < |z-1| < 1$.

18. a) Give an example for a power series which is convergent only at its center.

    b) Prove that every power series $\sum_{n=1}^{\infty} a_n (z-z_0)^n$ converges at its center $z = z_0$.

    c) Prove that a power series $\sum_{n=1}^{\infty} a_n (z-z_0)^n$ converges at a point $z = z_1 \neq z_0$, is converges absolutely for every z closer to z_0 than z_1.

19. Evaluate $\oint_C \frac{e^{-z^2}}{\sin 4z} dz$, where C is the unit circle in counter clockwise.

20. Prove that if f(z) is analytic and has a pole at $z = z_0$ then $|f(z)| \to \infty$ as $z \to z_0$ in any manner. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Show that the function $f(z) = 2x^2 + y + i(y^2 - x)$ satisfy the Cauchy – Riemann equation on the line y = 2x. Is it analytic on the line y = 2x ? Justify your answer.

    b) Prove that $\tanh^{-1} z = \frac{1}{2} \ln \frac{1+z}{1-z}$.

22. a) State and prove Cauchy – Riemann Equations.

    b) Find the principal value of $(2i)^{2i}$.

23. a) State and prove Cauchy's Integral formula.

    b) Evaluate $\oint_C \frac{z}{z^2 + 4z + 3} dz$, where C is the circle with center –1 and radius 2 in counter clockwise.

24. a) State and prove Cauchy's Inequality.

    b) Evaluate $\oint_C \frac{e^z}{(z-1)^2 (z^2+4)^2} dz$, for any contour C for which 1 lies inside and ±2i lie outside taken in counter clockwise. **(2×6=12)**