



K22U 2321

Reg. No. : .....

Name : .....



V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/  
Improvement) Examination, November 2022

(2019 Admission Onwards)

CORE COURSE IN MATHEMATICS

5B06MAT : Real Analysis – I

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**.

1. Determine the set A of all real numbers x such that  $2x + 3 \leq 6$ .
2. Let  $S = \left\{ 1 - \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$ . Find  $\inf S$  and  $\sup S$ .
3. State monotone convergence theorem.
4. State alternating series test.
5. Prove that signum function  $sgn$  is not continuous at 0.

PART – B

Answer **any 8** questions from among the questions **6** to **16**. These questions carry **2** marks **each**.

6. Find all  $x \in \mathbb{R}$  that satisfy  $|x + 1| + |x - 2| = 7$ .
7. State and prove triangle inequality.
8. If  $x \in \mathbb{R}$ , prove that there exists  $n \in \mathbb{N}$  such that  $x < n$ .
9. State and prove squeeze theorem.
10. Let  $(x_n)$  be a sequence of positive real numbers such that  $L = \lim \frac{x_{n+1}}{x_n}$  exists. If  $L < 1$ , prove that  $(x_n)$  converges and  $\lim(x_n) = 0$ .

P.T.O.

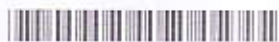


11. Prove that a Cauchy sequence of real numbers is bounded.
12. Prove that the sequence  $\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$  is divergent.
13. Prove that  $\sum_{n=0}^{\infty} r^n$  is convergent if  $|r| < 1$  and divergent if  $|r| \geq 1$ .
14. Prove that  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$  is divergent.
15. Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ .
16. State and prove sequential criterion for continuity.

## PART – C

Answer **any 4** questions from among the questions **17 to 23**. These questions carry **4 marks each**.

17. Let  $S$  be a subset of  $\mathbb{R}$  that contains atleast two points and has the property if  $x, y \in S$  and  $x < y$ . Prove that  $[x, y] \subseteq S$ .
18. Let  $(x_n)$  and  $(y_n)$  be sequences of real numbers that converge to  $x$  and  $y$  respectively. Prove that  $(x_n y_n)$  converges to  $xy$ .
19. Let  $e_n = \left(1 + \frac{1}{n}\right)^n$  for  $n \in \mathbb{N}$ . Prove that  $(e_n)$  is convergent.
20. Show that  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{1}{4}$ .
21. State and prove Ratio test.
22. Prove that  $g(x) = \sin \frac{1}{x}$  is continuous at every point  $c \neq 0$ .
23. State and prove boundedness theorem.



PART – D

Answer **any 2** questions from among the questions **24** to **27**. These questions carry **6** marks **each**.

24. a) State and prove nested interval property.

b) Prove that  $\mathbb{R}$  is not countable.

25. a) Prove that every contractive sequence is convergent.

b) Let  $f_1 = f_2 = 1$  and  $f_{n+1} = f_n + f_{n-1}$ . Define  $x_n = \frac{f_n}{f_{n+1}}$ . Prove that

$$\lim x_n = \frac{-1 + \sqrt{5}}{2}.$$

26. a) State and prove integral test.

b) Let  $a$  and  $b$  be two positive numbers. Prove that  $\sum (a + b)^{-p}$  converges if  $p > 1$  and diverges if  $p \leq 1$ .

27. State and prove maximum minimum theorem.

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