K24U 0060

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B12 MAT : Numerical Methods, Fourier Series and Partial Differential Equations

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **any four** questions out of **five** questions. **Each** question carries **one** mark. (4×1=4)

1. Define an even function and give an example.
2. Define Newton's divided difference interpolation polynomial.
3. Perform 2 iterations of Picard's method to find an approximation solution of the initial value problem $y' = x + y^2$, $y(0) = 1$.
4. Find Half Range cosine series for $f(x) = x^2$ in $0 \le x \le \pi$.
5. Write Laplacian equation in polar coordinates.

PART – B

Answer **any eight** questions out of **eleven** questions. **Each** question carries **two** marks. (8×2=16)

6. Solve $u_{xy} = -u_x$.
7. Find the unique polynomial p(x) of degree 2 or less such that p(1) = 1, p(3) = 27 and p(4) = 64 using Lagrange interpolation formula.

P.T.O.

8. Write the normal form of the equation $AU_{xx} + 2BU_{xy} + CU_{yy} = F(x, y, U, U_x, U_y)$.

9. Prove that $\mu^2 = 1 + \frac{1}{4}\delta^2$.

10. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series in the interval $0 \leq x \leq 2\pi$.

11. Determine the value of y when x = 0.1 given that $y(0) = 1,\ y' = x^2 + y,\ h = 0.05$.

12. Solve $\frac{dy}{dx} = 1 + xy$ with y(0) = 0 up to 3rd approximation by Picard's method of successive approximation.

13. Develop the Fourier series of $f(x) = x^2$ in $-2 \leq x \leq 2$.

14. Given $\frac{dy}{dx} = 1 + y^2$ where y = 0. When x = 0 find y(0.2).

15. Using the table find f as a polynomial in x,

x	−1	0	3	6	7
f(x)	3	−6	39	822	1611

16. Use Euler method to solve $\frac{dy}{dx} = x + xy + y,\ y(0) = 1$. Compute y at x = 0.15 by taking h = 0.15.

PART – C

Answer **any four** questions out of **seven** questions. **Each** question carries **four** marks. **(4×4=16)**

17. From the Taylor series for y(x) find y(0.1) correct to 4 decimal places if y(x) satisfies $y' = x - y^2$ and $y(0) = 1$.

18. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{1 + y^2}$ with initial condition y = 0 when x = 0. Use Picard's method to obtain y for x = 0.25, 0.5 and 1.0, correct to 3 decimal places.

19. Using Lagrange's interpolation formula, find the form of the function y(x) from the following table :

x	0	1	3	4
y	– 12	0	12	24

20. Find the fourier series of the periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$.

21. Find the temperature u(x, t) in a laterally insulated copper bar 80 cm long. If the initial temperature is $100 \sin\left(\frac{\pi x}{80}\right)$ °C and the ends are kept at 0°C, how long will it take for the maximum temperature in the bar to drop to 50°C ?

Physical data for copper : Density = 8.9 g/cm^3, Specific heat = 0.092 cal/g°C, thermal conductivity = 0.95 cal/cm sec.

22. Using Newton's forward difference formula, find the sum $s_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

23. Values of x (in degrees) and sin x are given in the following table :

x (in degree)	sin x
15	0.2588190
20	0.3420201
25	0.4226183
30	0.5
35	0.5735764
40	0.6427876

Determine the value of sin 38°.

PART – D

Answer **any two** questions out of **four** questions. **Each** question carries **six** marks. **(2×6=12)**

24. Derive D'Alembert solution of wave equation.

25. A sinusoidal voltage E sin ωt where t is time, is passed through a half wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function $u(t) = \begin{cases} 0 & \text{if } -L < t < 0 \\ E\sin\omega t & \text{if } 0 < t < L \end{cases}$

$p = 2L = \frac{2\pi}{\omega}, L = \frac{\pi}{\omega}$.

26. Using Runge-Kutta method of fourth order find y(0.2) from the initial value problem

$\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$ taking h = 0.2.

27. From the following table values of x and y determine :

i) f(0.23)

ii) f(0.29)

x	f(x)
0.20	1.6596
0.22	1.6698
0.24	1.6804
0.26	1.6912
0.28	1.7024
0.30	1.7139