K22U 0416

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the equation of the line passing through the points (3, –2, 4) and (–5, 7, 1).
2. State true or false : A set consisting of a single non zero vector is linearly dependent.
3. Define the formula for the linear transformation that rotates a vector (a_1, a_2) in $\mathbb{R}^2$ counter clockwise through an angle θ.
4. State Dimension Theorem.
5. What is the smallest possible nullity of a 3×5 matrix ?

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Let $S = \{(a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$. Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 - b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$. Is S a vector space ? Justify your answer.
7. Show that the set of all $n \times n$ matrices having trace equal to zero is a subspace of $M_{n \times n}(F)$.
8. Let W be a subspace of a vector space over a field F. Then prove that $v_1 + W = v_2 + W$ iff $v_1 - v_2 \in W$.
9. Check whether the set {(1, –1, 2), (1, –2, 1), (1, 1, 4)} is linearly independent or not.

10. Let S be a linearly independent subset of a vectorspace V. Then prove that there exist a maximal linearly independent subset of V that contains S.

11. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that T(1, 1) = (1, 0, 2) and T(2, 3) = (1, –1, 4), then find T(8, 11).

12. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing to row echelon form.

13. Let V and W be vector spaces and $T : V \to W$ be linear. Show that the nullspace N(T) and range of T, R(T) are subspaces of V and W respectively.

14. If A is a square matrix with λ as an eigenvalue, then prove that λ^{-1} is an eigenvalue of A^{-1}.

15. Find A^{-1} using Cayley Hamilton Theorem for the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

16. For what value of k the following system of homogeneous equations have a non trivial solution : $x + 2y - 3z = 0$, $2x + y + z = 0$ and $x - y + kz = 0$?

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Let V be a vector space. Then show that a subset W of V is a subspace of V if and only if the following conditions hold.
    a) $0 \in W$
    b) $x + y \in W$
    c) $cx \in W$.

18. If W_1 and W_2 are subspaces of a vectorspace V, then prove that $W_1 + W_2$ is a subspace of V that contains both W_1 and W_2.

19. Determine whether the set {(–1, 3, 1), (2, –4, –3), (–3, 8, 2)} is a basis for $\mathbb{R}^3$.

20. Find the matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$ with respect to the standard ordered basis.

21. Reduce to normal form and find the rank of $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$.

22. Solve

$x + 3y + 3z = 1$

$2x + 6y + 9z = 5$

$-x - 3y + 3z = 5.$

23. Find the null space and nullity of $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Show that the set of all $m \times n$ matrices with entries from a field F is a vector space over F.

25. Define basis of a vectorspace with an example. Show that every vectorspace of finite dimension has the same number of vectors.

26. Find the inverse of $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ using elementary row operations.

27. Find the null space and range space of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{pmatrix}$.